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CALCULATION OF METAL PLATE FUSION BY A CONCENTRATED ENERGY FLUX

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The one-dimensional problem of heating and melting of a metal plate by a constant surface heat source is considered. The kinetics of fusion front motion are studied with consideration of absorption of the latent heat of phase transition up to temperatures close to the boiling point.

Studies of the processes of heating and formation of a melt under the action of concentrated energy sources on condensed media have been under way for quite some time. Interest therein has been stimulated by the need to develop laser, plasma, electron-beam, ion, and other forms of materials processing. In such technological processes as laser doping, surfacing, laser-plasma compound synthesis, etc. redistribution of components initially deposited on the target surface, gas saturation, chemical compound synthesis, and other processes take place in the liquid phase. To study and optimize the latter it is necessary to know the depth of the melt pool and the temperature distribution therein to a sufficient accuracy. In connection with this, a series of studies [1-9] has been dedicated to solution of the problem of fusion under the action of a concentrated energy source. In [8] the approximate Biot method was used to consider fusion of a semi-infinite target. The shortcomings of that technique are: the complexity of theoretical justification, insufficient accuracy (the error in determining pool depth reaches 15%), and the absence of any generalization to fusion of finite plates. Numerical calculation by a computer was used in [9] with a finite difference technique and explicit specification of the fusion front. The shortcoming of this method is the necessity of composing a complex program.

In the present study we will offer an approximate analytical solution of the problem of heating and fusing a metal plate of finite thickness, which is characterized by simplicity, high accuracy (error of about 1%), and ease of use. Major attention will be given to phase transition kinetics.

We will briefly describe the process to be considered. A constant energy flux is incident on a metallic target of finite thickness and is absorbed upon the surface. We will assume that the coefficient for absorption of the concentrated energy flux by the surface is approximately constant, which is valid, for example, for a low-energy electron beam. Moreover, we let the transverse dimension of the source action zone R be much greater than the target thickness H: $R \gg H$. The problem can then be considered in one-dimensional formulation.

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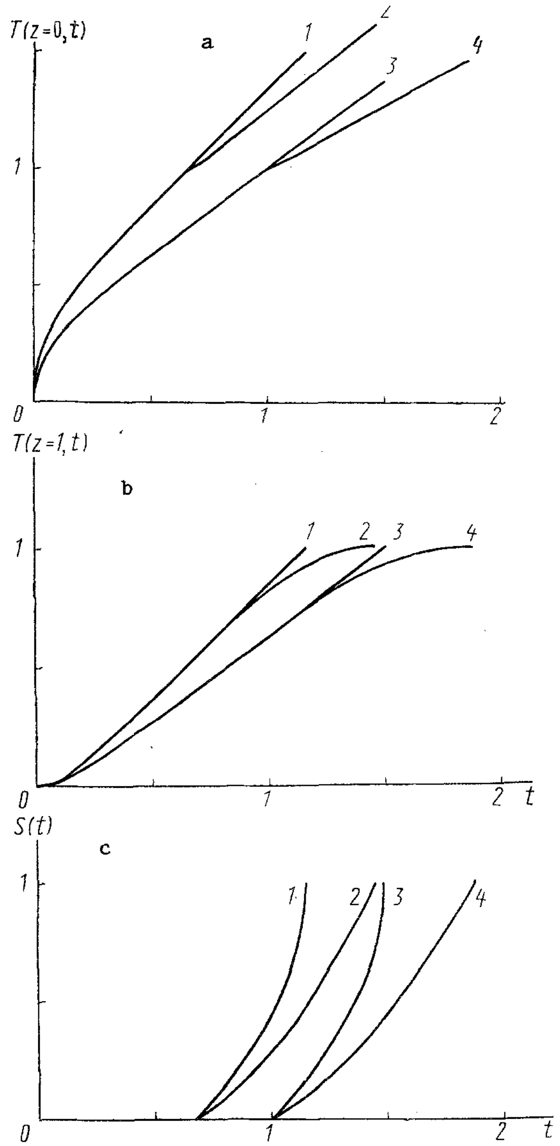

Fig. 1

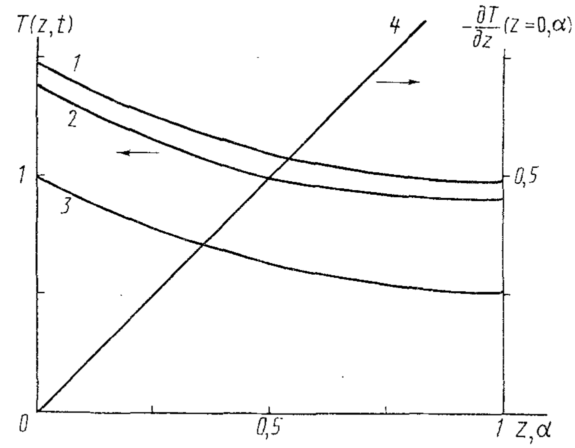

Fig. 2

Fig. 1. Temperature of front (a) and rear (b) target surfaces and melt depth (c) vs time for various parameter values: 1) $\beta = 0$, $\alpha = 1$; 2) 0.23 and 1; 3) 0 and 0.75; 4) 0.23 and 0.75.

Fig. 2. Spatial dependence of temperature field T(z, t) at various times: 1) $\beta = 0$, $\alpha = 1$, $t = 1.14$; 2) 0.23, 1, and 1.14; 3) 0, 1, and 0.66; 4) temperature gradient on front target surface $\partial T(z = 0,t)/\partial z$ vs α.

We also will assume that the power absorbed from the source $q \geq 10^9$ W/m^2, and will trace the action of the source up to the time when the target surface temperature reaches the metal boiling point or the target melts through. Then, as corresponding estimates will show, thermal losses due to convection, thermal radiation, and metal evaporation may be neglected in comparison to q. After the target is heated to the metal fusion point a melt layer is formed, the internal border of which, separating liquid and solid phases, moves into the depths of the target, while the outer boundary remains fixed. On this moving fusion front absorption of the latent heat of phase transition occurs. Assuming the thermophysical characteristics of the metal and its melt identical and constant, we obtain the following mathematical formulation:

$$a\frac{\partial^2 T}{\partial z'^2} = \frac{\partial T}{\partial t'},\quad 0 \leqslant z' \leqslant H,\quad t' \geqslant 0,\quad T(t' = 0,\ z') = T_0,$$

$$-\lambda \frac{\partial T}{\partial z'}\bigg|_{z'=0} = q,\quad a = \frac{\lambda}{c\gamma},\quad \frac{\partial T}{\partial z'}\bigg|_{z'=H} = 0;$$

$$t' \geqslant t_m: \quad -\lambda \frac{\partial T}{\partial z'}\bigg|_{z'=s(t')-0} = -\lambda \frac{\partial T}{\partial z'}\bigg|_{z'=s(t')+0} + \gamma L \frac{ds(t')}{dt'},$$

$$T(t', z' = s(t')) = T_m, \quad s(t \leqslant t_m) = 0,$$

$$T(t', z' = 0) \leqslant T_b, \quad T(t', z' = H) \leqslant T_m.$$

We introduce the dimensionless variables:

$$z = \frac{z'}{H}, \quad t = \frac{t'a}{H^2}, \quad T = \frac{T - T_0}{T_m - T_0}, \quad s = \frac{s}{H}, \quad T_m = 1,$$

$$\alpha = \frac{qH}{\lambda (T_m - T_0)}, \quad \beta = \frac{L}{c(T_m - T_0)}, \quad T_b = \frac{T_b - T_0}{T_m - T_0}.$$

Now, making use of the linearity of the thermal conductivity equation, we represent the unknown temperature field T in the form of the sum of the temperature field T_1 produced by the concentrated source alone and the temperature field T_2 produced by absorption of the heat of phase transition on the moving fusion front. $T = T_1 + T_2$. Then for the fields T_1 and T_2 the problem may be written as:

$$\frac{\partial^2 T_1}{\partial z^2} = \frac{\partial T_1}{\partial t}, \quad 0 \leqslant z \leqslant 1, \quad t \geqslant 0, \quad T_1(0, z) = \frac{\partial T_1}{\partial z}\bigg|_{z=1} = 0,$$

$$-\frac{\partial T_1}{\partial z}\bigg|_{z=0} = \alpha, \quad \frac{\partial T_1}{\partial z}\bigg|_{z=s(t)-0} = \frac{\partial T_1}{\partial z}\bigg|_{z=s(t)+0},$$

$$t \geqslant t_m: \quad \frac{\partial^2 T_2}{\partial z^2} = \frac{\partial T_2}{\partial t}, \quad 0 \leqslant z \leqslant 1, \quad T_2(t \leqslant t_m, z) = 0,$$

$$\frac{\partial T_2}{\partial z}\bigg|_{z=0} = \frac{\partial T_2}{\partial z}\bigg|_{z=1} = 0,$$

$$-\frac{\partial T_2}{\partial z}\bigg|_{z=s(t)-0} = -\frac{\partial T_2}{\partial z}\bigg|_{z=s(t)+0} + \beta \frac{ds(t)}{dt},$$

$$s(t \leqslant t_m) = 0, \quad T_1(t, z = s(t)) + T_2(t, z = s(t)) = 1,$$

$$T_1(t, 0) + T_2(t, 0) \leqslant T_b, \quad T_1(t, 1) + T_2(t, 1) \leqslant 1.$$

It is thus evident that T_1 is a solution of the problem of plate heating and fusion for $\beta = 0$, while T_2 is completely determined by the law of motion of the fusion front $z = s(t)$, which is given by the equation $T_1(t, s) + T_2(t, s) = 1$. In the final outcome T_2 depends indirectly (in terms of s(t)) on T_1.

We seek the temperature field T_1, knowing its asymptote as $t \to 0$ and $t \to \infty$, in the following form:

$$T_1 = \alpha (f_0(t) \exp(-f_1(t) z - f_2(t) z^2) + f_3(t) z^2).$$

Then, satisfying the boundary conditions at $z = 0$ and $z = 1$, we find:

$$f_3 = \frac{1}{2}(1 + 2 f_0 f_2) \exp(-f_1 - f_2), \quad f_1 = \frac{1}{f_0}.$$

Expanding T_1 near $z = 0$ in powers, substituting in the thermal conductivity equation, and equating terms with zeroeth and first powers of z, we obtain: $f_2 = 1/(6f_0^2)$ and a differential equation for f_0. To find f_0 the simplest approach is to merge its known asymptotes as $t \to 0$ and $t \to \infty$, representing the unknown function in the following form:

$$f_0(t) = \frac{2\sqrt{t}}{\sqrt{\pi}} \left(1 - \exp\left(-\frac{\beta_1}{\sqrt{t}} - \frac{\beta_2}{t} - \frac{\beta_3}{t\sqrt{t}}\right)\right)^{-1},$$

$$\beta_1 = \frac{2}{\sqrt{\pi}}, \quad \beta_2 = \frac{\beta_1^2}{2}, \quad \beta_3 = \beta_1(\beta_1^2 - 1)\frac{1}{3}.$$

The accuracy of the expression presented for f_0 is approximately 1% of the dimensionless fusion temperature, equal to unity.

To find the temperature field T_2 we solve an intermediate problem, having determined the temperature field T_3 produced by heating of a semi-infinite space by a heat source moving at constant velocity ϑ, the power density of which is equal to the same velocity ϑ:

$$\frac{\partial^2 T_3}{\partial z^2} = \frac{\partial T_3}{\partial t},\ z \geqslant 0,\ t \geqslant t_m,\ T_3(t \leqslant t_m,\ z) = 0,$$

$$\left.\frac{\partial T_3}{\partial z}\right|_{z=0} = T_3|_{z=\infty} = 0,\quad \vartheta(t-t_m) \leqslant 1,$$

$$\left.\frac{\partial T_3}{\partial z}\right|_{z=\vartheta(t-t_m)-0} = \left.\frac{\partial T_3}{\partial z}\right|_{z=\vartheta(t-t_m)+0} + \vartheta.$$

We seek the temperature field T_3 in the form

$$0 \leqslant z \leqslant \vartheta(t-t_m):\quad T_3 = h_0(t) + h_1(t)\,y + h_2(t)\,y^2,$$

$$z \geqslant \vartheta(t-t_m):\quad T_3 = h_0(t)\exp(-g_1(t)\,y - g_2(t)\,y^2),\quad y = z - \vartheta(t-t_m).$$

Using the thermal source of [4], we find:

$$h_0 = \int\limits_{t_m}^{t} \frac{dt'\vartheta}{2\sqrt{\pi(t-t')}} \exp\left(-\frac{\vartheta^2(t-t')}{4}\right) + \int\limits_{t_m}^{t} \frac{dt'\vartheta}{2\sqrt{\pi(t-t')}} \exp\left(-\frac{\vartheta^2(2(t-t_m)-(t-t'))^2}{4(t-t')}\right) \approx$$

$$\approx a_1 a_4 x\,(\exp(-x^2) + \sqrt{a_2\exp(-2x^2) + a_3 x^2})^{-1} + a_4 x \exp(-x^2)(1 + b_2 x + b_3 x^2)^{-1},$$

$$x = \frac{\vartheta\sqrt{t-t_m}}{2},\quad a_4 = \frac{2}{\sqrt{\pi}},\quad a_1 = \left(1 - \frac{3}{2\pi}\right)^{-1},$$

$$a_2 = (a_1-1)^2,\ a_3 = \frac{4a_1^2}{\pi},\ b_2 = 2\sqrt{\pi},\ b_3 = 4\pi - \frac{26}{3}.$$

Now, satisfying the boundary conditions and the thermal conductivity equation at $z = \vartheta(t - t_m)$, we obtain:

$$h_2 = \frac{h_1}{2\vartheta(t-t_m)},\quad h_1 = \frac{\vartheta(t-t_m)\,h_0'(t)}{1+\vartheta^2(t-t_m)},$$

$$g_1 = \frac{\vartheta - h_1}{h_0},\quad g_2 = g_1^2 - \vartheta g_1 - \frac{h_0'(t)}{h_0(t)}.$$

Then the temperature field T_2 for $\vartheta \geq 1$ can be represented approximately as:

$$0 \leqslant z \leqslant s(t):\ T_2 = -\beta\,(h_0 + h_1 y_1 + h_2 y_1^2 + h_0\exp(-y_2 g_1 - y_2^2 g_2)),$$

$$s \leqslant z \leqslant 1:\ T_2 = -\beta\,(h_0\exp(-y_1 g_1 - y_1^2 g_2) + h_0\exp(-y_2 g_1 - y_2^2 g_2)),$$

$$\vartheta = \frac{s(t)}{t-t_m},\quad y_1 = z - s(t),\quad y_2 = 2 - z - s(t).$$

Appropriate estimates will show that the accuracy of this representation of T_2 is about 1% of the dimensionless fusion temperature. It is considered here that the parameter β for metals varies over the interval 0.1-0.3. For titanium, for example, $\beta = 0.23$.

The temperature field T_2 found in this way still contains the unknown melt pool thickness s(t). This latter can be found from an equation specifying the equality of the temperature on the phase transition front to the melting point:

$$T_1(t,\ z=s(t))+T_2(t,\ z=s(t))=1,\quad \text{i.e.,}$$

$$\alpha\,(f_0\exp(-sf_1-s^2f_2)+s^2f_3)-\beta h_0(1+\exp(-2(1-s)g_1-4(1-s)^2g_2))=1.$$

The proposed method was used to calculate melting of a titanium plate for parameter values $\alpha = 1$ and 0.75, $\beta = 0$ and 0.23. In all these cases melt-through of the target occurred before the target surface was heated to the boiling point, equal in dimensionless units to ∿ 2. Figure 1a shows time dependences of temperature of the target front face. All the graphs break off at the time when the temperature of the target rear surface reaches the melting point of the metal. All physical quantities in the figures are de-dimensionalized as indicated above. It is evident from Fig. 1 that consideration of absorption of the heat of phase transition produces an inflection in the time dependence of front surface temperature at the moment when fusion begins, and increases the time required for target melt-through somewhat. The decrease in temperature for one and the same time reaches 10% of the fusion temperature.

As is evident from Fig. 1b consideration of absorption of the heat of phase transition causes the rate of increase of target rear surface temperature to vanish at the time melt-through is completed.

Figure 1c shows that if we neglect absorption of the heat of phase transition, the rate of increase in melt thickness at the moment of plate melt-through becomes infinite. At this time the thickness of the melt layer in the case where $\beta = 0.23$ is approximately half as much as for $\beta = 0$. It is also evident that for $\beta = 0.23$ the time dependence of target melt depth is almost linear, as was assumed in the calculations.

Figure 2 shows the spatial dependence of the temperature field at various times and the dependence of the temperature gradient on the front target surface upon dimensionless thermal flux power density α. It is evident from the figure that upon consideration of the absorption of the heat of phase transition the characteristic inflection of the temperature field $T(t, z)$ on the fusion front is hardly noticeable for the parameter values used.

To summarize, we may conclude that consideration of the absorption of the heat of phase transition in solving the problem of metallic plate fusion introduces a small correction to the temperature field (up to 10% of the fusion temperature) and a significant correction to the depth of the melt pool (up to 50% of the target thickness). Moreover, the behavior of $T(t, z = 1)$ and $s(t)$ at the moment melt-through is completed changes not only quantitatively, but also qualitatively.

NOTATION

t', t, source action time on target; t_m, time when target melting begins; R, transverse dimension of source action on target; H, target thickness; z', z, spatial coordinate measured inward into target from front surface; T_0, initial target temperature; T_m, target fusion temperature; T_b, target boiling point; T, T_1, T_2, T_3, temperature fields within target; a, thermal diffusivity coefficient; λ, thermal conductivity coefficient; γ, target density; c, specific heat; L, specific heat of target fusion; $s(t)$, target fusion depth; ϑ, α, β, dimensionless parameters of problem.

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